**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans:**

P(X>50) = 1 - (x <= 50)

Convert 50 to Z score

Z = (x - *μ) / σ* = (x - 45) / 8

P(X <= 50) = P( Z <= (50 - 45) / 8)

= Pr( Z <= 0.625) = 0.7323

= 73.23%

Probability that the service manager cannot meet his commitment = 100 - 73.23

= 26.76% = 0.2676

Ans : B. 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.

**Ans:**

1. More employees at the processing center are older than 44 than between 38 and 44.

Z = (x - *μ) / σ* = (x - 38) / 6

Pr( x <= 44) = P( Z <= (44 - 38) / 6)

= Pr( Z <= 1)

= 0.84134 = 84.134%

Probability of employees will be greater than 44 = 100 - 84.134

= 15.866

Probability of employees between 38 and 44 = Pr( x <= 44) - Pr( x >= 38)

Pr( x <= 44) = 0.84134

Pr( x >= 38) = 0.5

Pr( x <= 44) - Pr( x >= 38) = 0.84134 - 0.5

= 34.134%

The statement is true

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans:**

Probability of employees is less than

30 = Pr( X < 30)

Z = (x - *μ) / σ* = (30 - 38) / 6

Pr( X < 30) = P( Z <= (30 - 38) / 6)

= Pr( Z <= 1.3333)

= .09176 = 9.176%

Employees under the age of 30 at the center = 400 \* 0.09176

= 36.68%

So the statement is true

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans:**

we know that

*X1* = N(µ, σ^2 )

*X*2 = N(µ, σ^2 )

So

*X1* + *X*2 = N(µ1 + µ2, σ1^2 + σ2^2 )

= N(2 u, 2σ ^2 )

Also

2 *X*1 = N(2 u,4 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans:**

Probability getting values between a and b = 0.99

Probability towards left from a = -0.005

Probability towards right from b = +0.005

μ = 20

*σ* = 100

Z = (x - *μ) / σ*

For the probability of 0.005, z value is -2.57

Z \* μ + *σ* = x

-(-2.57) \* 20 + 100 = 151.5

(-2.57) \* 20 + 100 = 48.5

Ans : D. 48.5, 151.5

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Mean profit = RS 540 Million

Std deviation = RS 225 Million

Range is RS State norms interval (0.95, 540, 225)

Range is RS 99.0081034 , 980.991896

1. Specify the 5th percentile of profit (in Rupees) for the company

X=μ + Zσ

μ = 540

σ = 225

Z = -1.645 (5th percentile)

X = 540(-1.645)\*225

= 169.87

1. Which of the two divisions has a larger probability of making a loss in a given year?

Probability of division 1 making a loss p(X<0) Stats. norm. cdf (0, 5, 3) = 0.0477903

Probability of division 2 making a loss p(X<0) Stats.norm. cdf(0, 7, 4)

= 0.0400591